COMPARISON OF NON-ELLIPTIC CONTACT MODELS: TOWARDS FAST AND ACCURATE MODELLING OF WHEEL-RAIL CONTACT

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Background

• Demands to investigate surface deteriorating phenomena
  → accurate contact patch and stress distr. prediction
• Online application of the contact model in MBS codes
  → limitations on computational time

Trade-off between accuracy and time efficiency

• Elliptic models:
  o Hertz, equivalent elastic
  o Non-physical patch estimation and inaccurate stress distr. in several contact cases
• Non-elliptic contact models:
  o Advanced models: using FEM or BEM
  o Fast models: simplified contact conditions
Fast non-elliptic contact models

- Analytical estimation of the contact patch
- Based on virtual interpenetration (VI)
- Tangential part based on FASTSIM (Kalker 1982)
- Well-known models:
  - Kik-Piotrowski (1996)
  - Linder (1997)
  - Ayasse-Chollet (2005) (STRIPES)
Virtual interpenetration

- The exact contact equation:

$$z(x, y) = \delta_0 - \bar{u}_z$$

- Assume a rigid interpenetration: \( \bar{u}_z = 0 \)

- Penetrate by a fraction of the prescribed pen.: \( \delta_v = \epsilon \delta_0 \)

- Solve the new equation to find the patch boundaries:

$$z(x, y) = Ax^2 + f(y_i) = \epsilon \delta_0$$
Linder method

- The scaling factor is set to a constant: $\epsilon = 0.55$
- In a Hertzian case, the semi-axes are:
  \[ A_0 x^2 + f(0) = \epsilon \delta_0 \Rightarrow x = a = \sqrt{0.55 \delta_0 / A_0} \]
  \[ b = \sqrt{0.55 \delta_0 / B_0} \]
- The Hertz solution is:
  \[ a_H = m_0 \sqrt{\frac{\delta_0}{r_0(A_0 + B_0)}} \quad b_H = n_0 \sqrt{\frac{\delta_0}{r_0(A_0 + B_0)}} \]
- The semi-axes ratio:
  \[ \frac{a}{b} = \frac{B_0}{A_0} \quad \frac{a_H}{b_H} = \frac{m_0}{n_0} \]
- Same results, only if: $B_0 = A_0$
Kik-Piotrowski method

- Same scaling factor but a correction is applied
- Axes ratio is set equal to Hertz’s:

\[
\frac{a_c}{b_c} = \frac{n_0}{m_0}, \quad a_c b_c = ab
\]

- The semi-axes are then:

\[
a_c = \sqrt{ab n_0/m_0} = 0.55 \delta_0 \sqrt{\frac{n_0}{m_0}} \frac{n_0/m_0}{A_0 B_0}
\]

\[
b_c = \sqrt{\frac{ab}{n_0/m_0}} = 0.55 \delta_0 \sqrt{\frac{m_0/n_0}{A_0 B_0}}
\]

- Area may still be different from Hertz’s!
STRIPES ($A$-correction)

- Scaling factor is a constant based on the geometry:
  \[
  \epsilon = \frac{n_0^2}{r_0(A_0 + B_0)} B_0
  \]

- Local curvatures at $y_i$ are corrected as well
- If only the long. curvature is corrected:
  \[
  A_{ci} = \left(\frac{n_i}{m_i}\right)^2 B_i
  \]

- Semi-axes are:
  \[
  a = m_0 \sqrt{\frac{\delta_0}{r_0(A_0 + B_0)}}
  \]
  \[
  b = n_0 \sqrt{\frac{\delta_0}{r_0(A_0 + B_0)}}
  \]
  
  - exactly the same as Hertz’s!
STRIPES (A&B-correction)

- Scaling factor is based on the geometry:
  \[ \epsilon = \frac{n_0^2}{r_0(1 + (n_0/m_0)^2)} \]

- Both curvatures are corrected:
  \[ A_{ci} = \frac{(A_i + B_i)(n_i/m_i)^2}{(1 + (n_i/m_i)^2)} \]
  \[ B_{ci} = \frac{(A_i + B_i)}{(1 + (n_i/m_i)^2)} \]

- Semi-axes are:
  \[ a = \sqrt{\epsilon \delta_0 / A_{c0}} = m_0 \sqrt{\frac{\delta_0}{r_0(A_0 + B_0)}} \]
  \[ b = n_0 \sqrt{\frac{\delta_0}{r_0(B_0 + B_0(n_0/m_0)^2)}} \]

  - Same length but different width from Hertz’s!

- Approaches Hertz’s when \( A_0/B_0 \to 0 \) or 1
Tangential part

• Tangential part is treated using FASTSIM

• FASTSIM is originally used for elliptical patches

• Equivalent ellipse:
  - Kik and Piotrowski (1996)
  - Defining an equivalent ellipse for each zone, using elliptical flexibility parameters

• Local ellipses for each strip:
  - Linder (1997), Ayasse and Chollet (2005)
  - Strip discretization of the patch
  - Assigning an ellipse into each strip and using its elliptical parameters
Local ellipse assignment

- **Linder method:**
  - All local ellipses have the same lateral semi-axis.

- **STRIPES method:**
  - Local ellipses are based on the local curvature values at the centre of the strip.
Case study

- Models are implemented using MATLAB
  - As documented in the literature
  - STRIPES model: $A$-correction approach

- A single wheel on rail example is solved:
  - Right wheel-rail pair
  - S1002/UIC60 (1:40)
  - Zero lateral displacement (central wheelset position)
  - Normal load: 78.5 kN
  - Spin= 0.052 rad/m (pure spin)

- CONTACT software (BEM) results are taken as reference
  - Since, it is bound to half-space assumption, this case study is confined to tread contact
Contact patch

Graph showing the contact patch with labels for CONTACT, STRIPES, Kik-Piotrowski, and Linder. The graph indicates the relationship between longitude and latitude, with a direction towards gauge-corner.
Contact pressure distribution

Maximum Pressure

Pressure [MPa]

Lat. [mm]

CONTACT

STRIPES

Kik-Piotrowski

Linder

Introduction

Theory

Results

Conclusions
Creep forces

<table>
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<tr>
<th>Model</th>
<th>Long. [N]</th>
<th>Lat. [N]</th>
<th>Tot. [N]</th>
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<td>Linder</td>
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<td>-1672</td>
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CONTACT: same spin throughout the patch

STRIPES: spin calculated at the center of each strip
Improving pressure distribution

• In STRIPES, negative $B$ values are not allowed.
  - The negative $B$ is replaced by a minimal positive value
  - Smoothing filter is then applied to achieve smooth patch boundaries

• Smoothing also affects the pressure values and smears out the peaks

• To avoid smoothing, a new correction of negative $B$ values is suggested
  - Instead of cutting negative $B$ values out, shift them upward
  - Avoid sharp changes in $B$

• Since $A$-correction is sensitive to low $B$ values, $A&B$-correction is also of interest.
Wheelset central position

Contact patch

Maximum Pressure

CONTACT

STRIPES

STRIPES (A&B-correction)
Off-set case: $\Delta y = -1 \text{ mm}$
Off-set case: $\Delta y = 1 \text{ mm}$
Off-set case: $\Delta y = 2$ mm

CONTACT

STRIPES

STRIPES (A&B-correction)
Conclusions

• Patch prediction by VI-based models should be improved in off-set cases

• Contact pressure distr. deviates from reference in wheelset central position

• The A&B-correction strategy and avoidance of smoothing leads to improved pressure distr. prediction

• Non-planar patch and spin variation considered by STRIPES. Pronounced effects in the gauge-corner contact
Future work

• Comparison of fast non-elliptic models to FEM results in gauge-corner contact

• Improve the contact patch estimation of models based on virtual interpenetration (in off-set cases)

• Investigate the non-elliptic model based on semi-Winkler approach (Telliskivi 2004)
Thanks for your attention!

Any questions or comments?

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